Seminar-/Project-Thesis | Language: English

Handling Irreversibility in Phase-Field Models for Brittle Fracture

Variational approaches for modelling crack propagation, also referred to as phase-field fracture models have come up as an attractive alternative to the conventional *discontinuous models* such as XFEM, see [1] for a recent review. A primary reason for such interest in this methodology is its suitability to be implemented in a standard, confirming finite element framework. In the phase-field approach, the discontinuous crack set is approximated by means of a continuous crack phase-field z, thereby resulting in a (coupled) nonlinear system of PDEs. The variational form of the quasi-static crack propagation problem then reads: find $z \in [0,1]$, $\mathbf{u} : \Omega \rightarrow \mathbb{R}^{\text{dim}}$, such that

$$\inf_{z,\,\mathbf{u}} \left\{ \underbrace{\int_\Omega [g(z)\Psi_+ + \Psi_-]\,\mathrm{dV}}_{\mathcal{U}} + \underbrace{\frac{G_c}{c_w}\int_\Omega \left[\frac{w(z)}{l_c} + l_c|\nabla|^2\right]\mathrm{dV}}_{\mathcal{G}} \underbrace{- \int_\Omega \mathbf{b}\cdot\mathbf{u}\,\mathrm{dV} - \int_{\partial\Omega} \mathbf{t}\cdot\mathbf{u}\,\mathrm{dA}}_{-\mathcal{W}^{\text{ext}}} \right\} \tag{1}$$

where the terms $\mathcal{U}$, $\mathcal{G}$ and $\mathcal{W}$ denote the total elastic strain energy, the fracture energy and the external work respectively.

Despite the rapid development of the phase-field fracture models in the previous decade, several aspects need to be explored in further detail. One such aspect is the *irreversibility* of crack propagation. In order to ensure that the crack doesn't heal during unloading, the minimization problem of (1) has to be supplemented with appropriate constraints on the crack phase-field. One such idea is to imply a *history function* [2] representing the maximum positive strain energy over all *previous* time-points. This thesis work shall aim at exploring an extension of the aforementioned approach by considering the history function at the *current* time-point there by resulting in a coupled system of equations, which we plan to solve using a *staggered scheme* [3]. The implementation shall be done in the existing `C++` source-code (based on `deal.ii` library) and comparisons with other existing approaches for suitability under different loading scenarios shall be made.

References

[1] L. De Lorenzis and T. Gerasimov. "Numerical Implementation of Phase-Field Models of Brittle Fracture". In: *Modeling in Engineering Using Innovative Numerical Methods for Solids and Fluids*. Ed. by L. De Lorenzis and A. Düster. Springer International Publishing, 2020, pp. 75–101.

[2] C. Miehe, M. Hofacker, and F. Welschinger. "A phase field model for rate-independent crack propagation: Robust algorithmic implementation based on operator splits". In: *Computer Methods in Applied Mechanics and Engineering* 199.45 (2010), pp. 2765–2778.

[3] M. Ambati, T. Gerasimov, and L. De Lorenzis. "A review on phase-field models of brittle fracture and a new fast hybrid formulation". In: *Computational Mechanics* 55.2 (2015), pp. 383–405.

Fields: Material Mechanics, Fracture Modeling

Required skills: Linear Continuum Mechanics, Linear Finite Elements, C++

Desirable skills: Nonlinear Finite Elements,texttt deal.ii, Fracture Mechanics

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