Griffith's criterion for sharp crack and phase field fracture

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Griffith's criterion models the propagation of sharp cracks in brittle material on the basis of the interplay between toughness and energy release. We will focus on the difference between steady state and unsteady state propagation, and on their properties as far as energy identity. Steady and unsteady evolutions are typical of rate-independent processes with non-convex energies; in fracture they are well illustrated in a couple of standard numerical examples: the double cantilever beam (DCB) and the single edge notch under tension (SENT). Next we will introduce the phase field approach. First we will discuss the conver- gence of the energies, in terms of Gamma-convergence, and then we will introduce a notion of energy release rate. We generate evolutions by means of "alternate min- imization", i.e., a time discrete staggered scheme, where irreversibility is modeled by monotonicity in time of the phase field parameter. In the steady state regime, the resulting evolution is simultaneous (in displacement and phase field parameter) and satisfies Griffith's criterion in terms of toughness and phase field energy release rate; in the unsteady regime it is instead staggered and Griffith's criterion does not hold, in general. Technically, the proof relies on the strong convergence of the order parameter and on Kuratowski convergence of (a suitable reparametrization of) the time discrete points. We will comment on the thermodynamic consistency of the irreversibility constraint, showing monotonicity of the dissipated energy. We will compare the numerical evolutions, computed with FreeFEM++, in the sharp crack and in the phase field setting, showing a good consistency of the two approaches. The differences between steady and unsteady evolutions will be illustrated also in terms of the numerical convergence of the staggered scheme. Finally, we will present partial results and open problems related to the system of PDEs and the convergence of energy release and evolutions from phase field to sharp crack, i.e., as the internal length vanishes.